PDA

$\delta$: $(q, a, A)\ (q', \alpha)$ $a \in \Sigma$, $a = \varepsilon$

$\alpha \in \Gamma^*$

Configurations (Instantaneous description:

$(q, w, \underline{\alpha})$

stack content

remaining input

current state

$$\big(\underline{(q, a, A)}\ (q', \beta)\big) \in \delta \quad a \in \Sigma$$

$$(q, aw, A\alpha) \longrightarrow (q', w, \beta\alpha)$$

$$\big((q, \varepsilon, A)\ (q', \beta)\big) \in \delta$$

$$(q, w, A\alpha) \longrightarrow (q', w, \beta\alpha)$$

$$c_1 \longrightarrow c_2 \longrightarrow c_3 - - - c_n$$

$$c_1 \xrightarrow{*} c_n$$

$$(q_0, w, \perp) \xrightarrow{*} (q_f, \varepsilon, \alpha)$$

$$L(A) = \{ w \mid (q_0, w, \perp) \xrightarrow{*} (q_f, \varepsilon, \alpha) \}$$

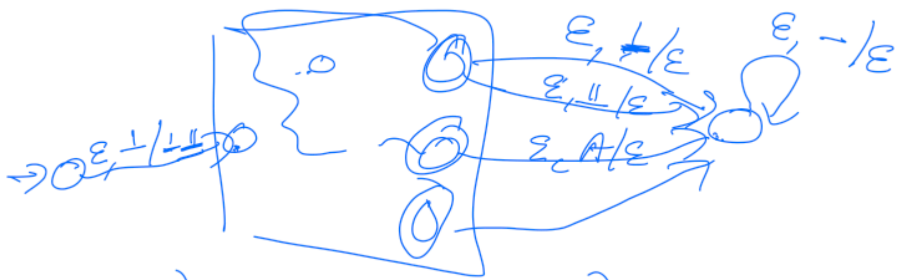
$q_f \in F$ }

## Acceptance by empty stack.

$$L(A) = \{ w \mid (q_0, w, \perp) \xrightarrow{*} (q, \varepsilon, \varepsilon) \}$$

<u>Claim</u>: Both acceptance mechanisms are equi-powerful.

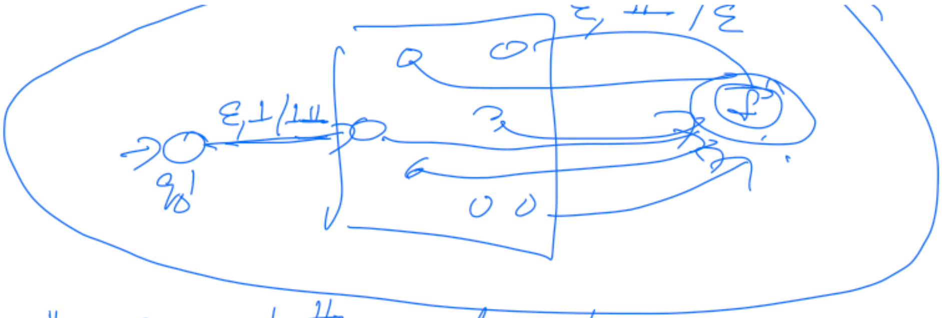
By final states $\rightsquigarrow$ empty stack.




$$(q_0, w, \perp) \rightarrow (q_0', w, \perp \perp\!\!\perp)$$


$\perp\!\!\perp$ : new bottom element

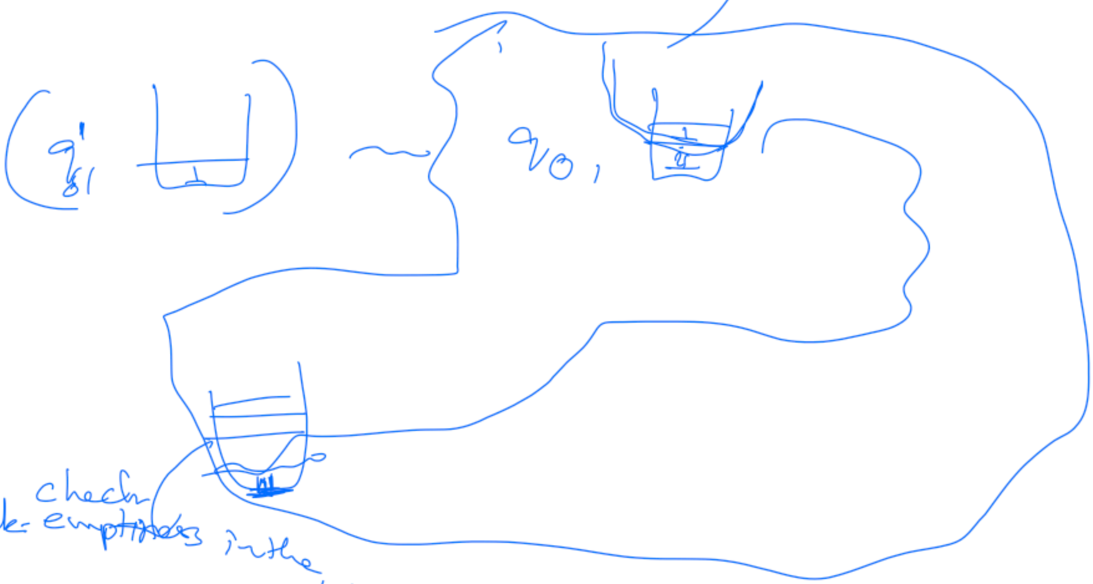
Empty stack $\rightsquigarrow$ final states




$\perp\!\!\perp$: new bottom element.

$\Gamma' = \Gamma \cup \{\perp\!\!\perp\}$

$(Q', \Sigma, \Gamma', q_0', \perp, \delta', \{f\})$




check stack emptiness in the old configuration

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~~Gram~~ "CFG = PDA"

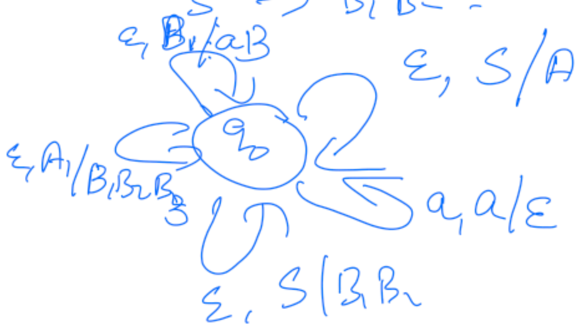
CFG $\longrightarrow$ PDA

$S \xrightarrow{*} a_1 a_2 A a_3 A B a_1 \ldots$

$\to \quad - \quad - .$ $\qquad \Gamma = N \cup \Sigma$

$S \to A_1 A_2 \dots A_k$

$S \to B_1 B_2 \dots$





$A_1 \to B_1 B_2 B_3$

$B_1 \to aB$

$\underline{B} a B_1$

$A \to BC$

$A \to \cancel{a} B_1 \dots B_k$

$(a, A / B_1 \dots B_k)$

ε-elimination possible

assume G is in Greibach NF

PDA $\to$ G

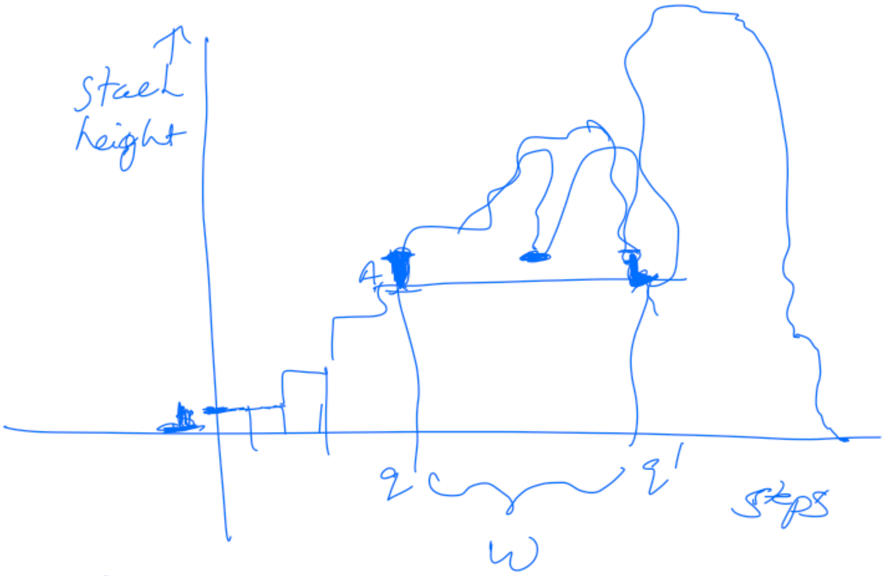
PDA $(Q, \Sigma, \Gamma, q_0, \bot, \delta)$

accepting by empty stack.

$L_{(qAq')}$

$[qAq']$

$q, A, q'$




$\bigcup_{q \in Q} [q_0 \perp q]$

$S \to [q_0 \perp q_1]$

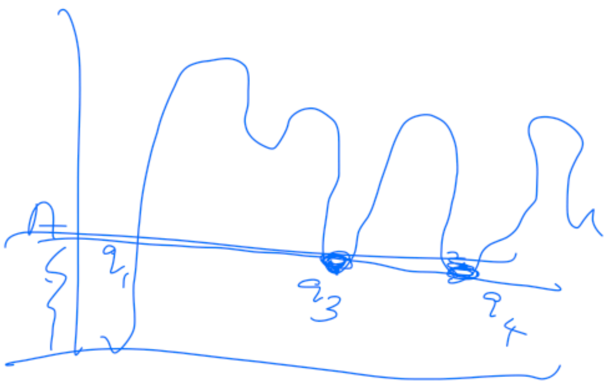
$S \to [q_0 \perp q_2]$

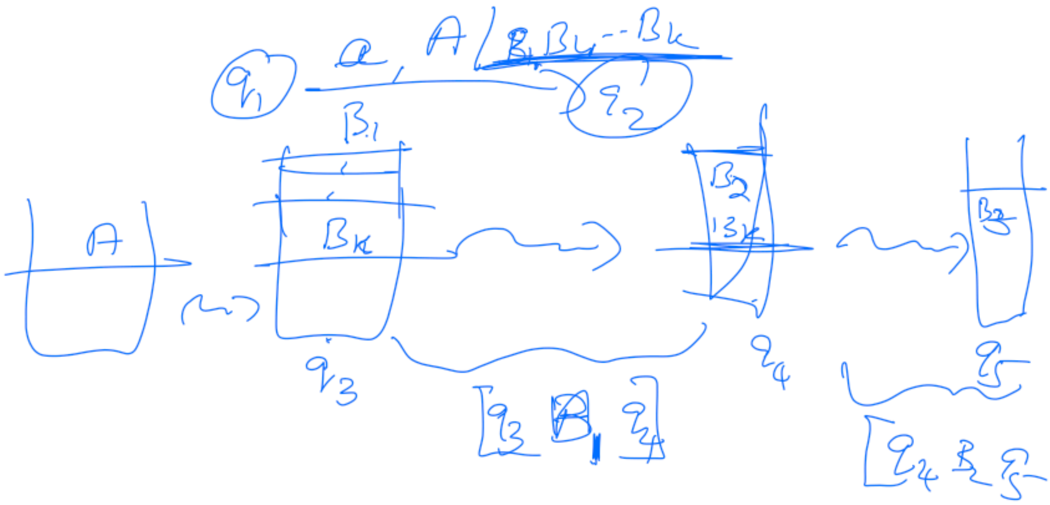
$\vdots$

$N = S \cup [qAq']$

$Q \times \Gamma \times Q$

$q_1 A q_2 \to a$



$$[q_1 A q_2] \to a\,[q\, B_1\, r_1][r_1\, B_2\, r_2][r_2\, B_3\, r_3] \ldots [r_{k-1}\, B_k\, q_2]$$

$$r_i \in Q$$

$$Q \times \Gamma \times Q \qquad [q_1 A q_2] \to a\,[q\, B_1\, r_1][r_1\, B_2\, r_2] \ldots [r_{k-1}\, B_k\, q_2]$$

$$q_1 \xrightarrow{a,\ A / B_1 \cdots B_k} q \qquad q,\ A$$

$$Q^k$$

for $q_1 A q_2 \in Q \times \Gamma \times Q$

{

  foreach $q_1 \xrightarrow{a,\ A / B_1 \cdots B_k} q \in \delta$

  {

    for each $r_1 \cdots r_k \in Q^k$

    {

    }

  }

}

>

$[q_1 A q_2] \to a [q_1 B q_2]$ if $k = 1$

Linear grammars are less powerful than CFG

$B \to aBa \mid bBb \mid \varepsilon \mid a \mid b$

Quadratic grammars are ~~as~~ enough to capture entire CFL (thanks to Chomsky NF)